Toward a unified description of hadro- and photoproduction

$S-$wave $\pi-$ and $\eta-$photoproduction amplitudes

M. Paris[GWU] D. Arndt[VPI] R. Workman[GWU]
W. Briscoe[GWU] I. Strakovsky[GWU]

[GWU]Data Analysis Center
Center for Nuclear Studies
The George Washington University

[VPI]Formerly of the
Department of Physics
Virginia Tech

12th International Conference on Meson-Nucleon Physics and the Structure of the Nucleon
June 2, 2010

Center for Nuclear Studies

Outline

CNS Center for Nuclear Studies

Outline

Center for Nuclear Studies

Scattering Analysis Interactive Database
Major initiatives

1. Phenomenology
   - Suite of FORTRAN programs to analyze $2 \to 2$ & 3 body scattering and reaction data developed primarily by Dick Arndt
   - Routines: database, fit, and analysis
   - Implement parameterizations via field theoretic constraints
   - Reactions: $\pi N \to \pi N, \eta N, \pi\pi N$; $KN \to KN$; $NN \to NN$; $\pi d \to \pi d$; $\pi d \to pp$; $\gamma N \to \pi N, \eta N, \eta' N, KY$; $eN \to e\pi N$
   - Open access
     Passwordless secure shell: `ssh -C -X said@said.phys.gwu.edu`
     Web interface: `http://gwdac.phys.gwu.edu`
     Web users $\sim$ 100/month: modelers, comparisons to data (measured and unmeasured), exp'l planning, simulations, event generators, detector design and callibation, ...
2. Theory & modeling
   - Dynamical multichannel effective field theory models
   - Comparison to phenomenological parameterizations
   - Derive guidance from phenomenology for approximations

CNS Center for Nuclear Studies

Outline

Center for Nuclear Studies

Motivation

for the current exploratory study

MP & R. Workman, Phys. Rev. C, in press; arXiv: 1004.0455v1

1. Require a model independent method to obtain partial wave amplitudes
2. Existing knowledge of resonances mostly from hadronic scattering/reactions: $\pi N \to \pi N, \pi N \to \eta N, \pi N \to \omega N$
   $\to$ complete $\pi N \to \pi N$ measurement over significant kinematic range
3. New precision electromagnetic facilities: Bonn, JLab, Lund, Mainz, ...
   $\to$ renaissance in reaction theory and resonance physics
   $\to$ quality & quantity of data rival/surpass hadronic data
   $\to$ possibility to 'back-constrain' hadronic amplitudes in *unitary* formalism
   $\to$ talk by *I. Strakovsky*, Session 6B – Crystal Ball @ MAMI-C
4. Extend SAID approach used in hadronic sector to electromagnetic
   - Hadronic sector $\pi N \to \pi N$ & $\pi N \to \eta N$ (untouched)
     4 channel Chew-Mandelstam approach $\{\pi N, \eta N, \pi\Delta, \rho N\}$
   - Electromagnetic sector $\gamma N \to \pi N$ & $\gamma N \to \eta N$
     Introduce 4 channel Chew-Mandelstam approach $\{\pi N, \eta N, \pi\Delta, \rho N\}$ with same hadronic "rescattering" matrix
5. Obtain $\eta-$photoproduction amplitude with *resonant* phase – various calculations [*Green & Wycech; Kaiser et. al.; Aznauryan*] yield wide range of phases
6. Study baryon resonances in ηN channel
7. Study η-sector physics

CNS Center for Nuclear Studies

Outline

Center for Nuclear Studies

Unitarity constraint on T
Threshold branch points

Parameterization (Chew-Mandelstam) adheres to analytic structure dictated by unitarity in the physical region $W > m_i + m_t$

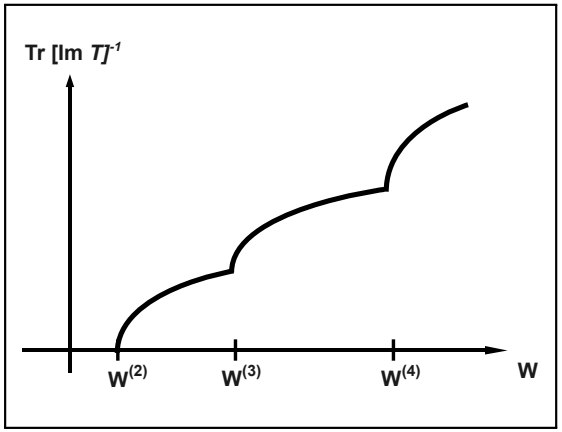
- S matrix

$$S_{\alpha\beta}(E) = \langle \mathbf{k}_\alpha \alpha | S | \mathbf{k}_\beta \beta \rangle$$
$$= \delta^{(3)}(\mathbf{k}_\alpha - \mathbf{k}_\beta)\delta_{\alpha\beta} + 2i\pi\delta(E_\alpha - E_\beta)\langle \mathbf{k}_\alpha \alpha | T | \mathbf{k}_\beta \beta \rangle$$

$E = W =$ center-of-mass energy; α, β channels $\pi N, \eta N, \pi\Delta, \rho N, \ldots$

- $S^\dagger S = SS^\dagger = 1 \implies$ constraint on T

$$T - T^\dagger = T^\dagger \rho T$$
$$\text{Im } T^{-1} = -\rho, \quad \rho_{\alpha\beta} = \delta_{\alpha\beta}\rho_\alpha$$
$$\rho_\alpha = \theta(W - m_{\alpha+})\frac{\pi k_\alpha E_{\alpha 1} E_{\alpha 2}}{W}$$



- Unitarity $\implies$ real branch points at normal thresholds
- Ignore non-analytic structure in $W < 0$ region (for now)

Center for Nuclear Studies

Outline

Center for Nuclear Studies

K-matrix and Chew-Mandelstam forms

after Basdevant & Berger, Phys. Rev. D 19 (1979); Babelon et. al. Nuc. Phys. B113 (1976)

Change normalization; matrix notation

K-matrix

$$\text{Im } T^{-1} = -\theta(W - M_+) \text{ a diagonal matrix}$$

$$T^{-1} = \text{Re } T^{-1} + i\text{Im } T^{-1} = K^{-1} - i\theta(W - M_+)$$

$$T = K + Ki\theta(W - M_+)T$$

Chew-Mandelstam

$$T^{-1} = (K^{-1} + \text{Re } C) - (\text{Re } C + i\theta(W - M_+)) = \overline{K}^{-1} - C$$

$$T = \overline{K} + \overline{K}CT$$

$$C_\alpha(W) = \frac{W - W_s}{\pi} \int_{W_t}^{\infty} dW' \frac{\rho_\alpha(W')}{(W' - W_s)(W' - W)}$$

$$K = \overline{K} + \overline{K}[\text{Re } C]K \implies K = \{1 - \overline{K}[\text{Re } C]\}^{-1}\overline{K}$$

Center for Nuclear Studies

Outline

Center for Nuclear Studies

N/D approach
Relation to C-M approach

$$T(W) = D^{-1}(W)N(W)$$

$$\begin{aligned} \operatorname{Im} D(W) &= N(W)\operatorname{Im} T^{-1}(W) & \operatorname{Im} N(W) &= 0 & W &> m_i + m_t \\ \operatorname{Im} N(W) &= D(W)\operatorname{Im} T(W) & \operatorname{Im} D(W) &= 0 & W &< 0 \end{aligned}$$

$$D(W) = \sum_{i=1}^{n_p} D(W; W_i) - \frac{1}{\pi}\prod_{i=1}^{n_p}(W - W_i)\int_{W_t}^{\infty} dW' \frac{N(W')\rho(W')}{(W' - W)\prod_j (W' - W_j)}$$

$$N(W) = K\left\{\sum_i D(W; W_i) - \frac{1}{\pi}\prod_{i=1}^{n_p}(W - W_i)\fint_{W_t}^{\infty} dW' \frac{N(W')\rho(W')}{(W' - W)\prod_j (W' - W_j)}\right\}$$

Chew-Mandelstam approximates N, neglecting left-hand cut

$$N(W) = \overline{K}(W)$$

$$\overline{K}_{\alpha\beta} = \sum_{n=0}^{n_{\alpha\beta}} c_{\alpha\beta,n}\overline{z}^n_{\alpha\beta}(W)$$

CNS Center for Nuclear Studies

Outline

Center for Nuclear Studies

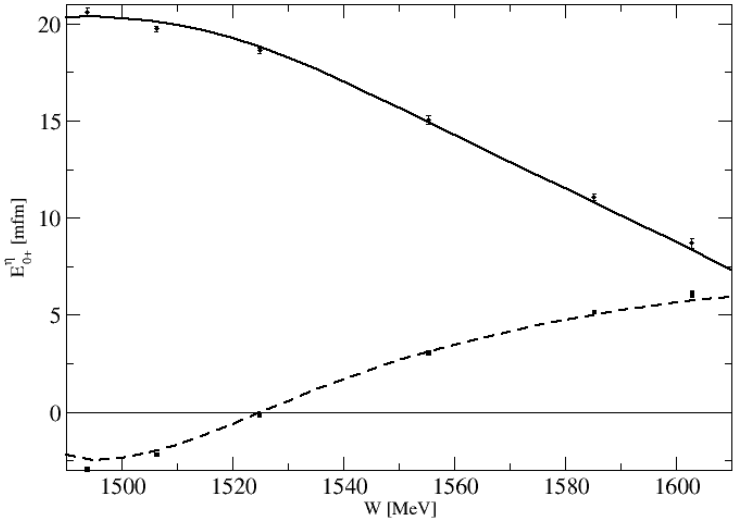
K-matrix formalism

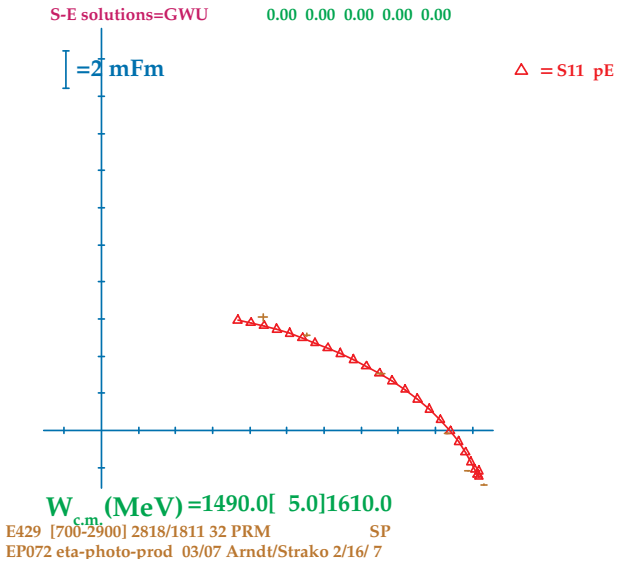
Two-channel formalism (trivially generalizable to N 2-body channels)

$$T_{\pi\gamma} = (1 + iT_{\pi\pi})K_{\pi\gamma} + iT_{\pi\eta}K_{\eta\gamma} \qquad T_{\eta\gamma} = (1 + iT_{\eta\eta})K_{\eta\gamma} + iT_{\eta\pi}K_{\pi\gamma}$$

Reduction via hadronic matrix to various forms

$$\begin{aligned} T_{\eta\gamma} &= \left(K_{\eta\gamma} - \frac{K_{\pi\gamma}K_{\eta\eta}}{K_{\pi\eta}}\right)(1 + iT_{\eta\eta}) + \frac{K_{\pi\gamma}}{K_{\pi\eta}}T_{\eta\eta} \\ &= A(W)(1 + iT_{\eta\eta}(W)) + B(W)T_{\eta\eta}(W) && \text{Form 1} \\ &= A'(W)(1 + iT_{\pi\pi}(W)) + B'(W)T_{\pi\pi}(W) && \text{Form 2} \end{aligned}$$





Anticipate resonant phase in region 1.49 GeV $\lesssim W \lesssim$ 1.6 GeV

Outline

Center for Nuclear Studies

Chew-Mandelstam parameterization

$\pi-$ & $\eta-$photoproduction

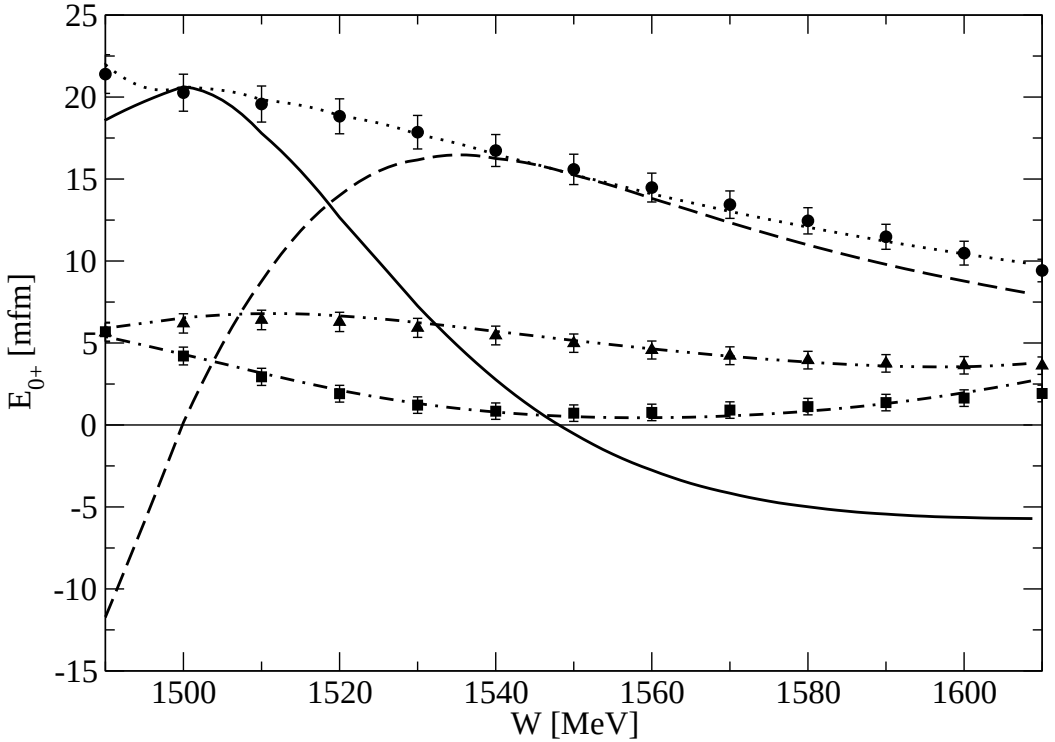
Current hadronic parameterization fits $\pi N \to \pi N, \pi N \to \eta N$, DR, ...

$$T_{\alpha\beta} = \sum_{\sigma} [1 - \overline{K}C]^{-1}_{\alpha\sigma} \overline{K}_{\sigma\beta}$$

Generalized to photoproduction (hadronic matrix fixed by above)

$$T_{\alpha\gamma} = \sum_{\sigma} [1 - \overline{K}C]^{-1}_{\alpha\sigma} \overline{K}_{\sigma\gamma}$$

Perform fit at amplitude level to $\mathrm{Re}\, E^{\pi}_{0+}$, $\mathrm{Im}\, E^{\pi}_{0+}$ & $|E^{\eta}_{0+}|$



Solid curve: $\mathrm{Re}\, E^{\eta}_{0+}$
Dashed curve: $\mathrm{Im}\, E^{\eta}_{0+}$
Dot-dashed curve: $\mathrm{Re}\, E^{\pi}_{0+}$
Double dot-dashed curve: $\mathrm{Im}\, E^{\pi}_{0+}$
Dotted curve: $|E^{\eta}_{0+}|$

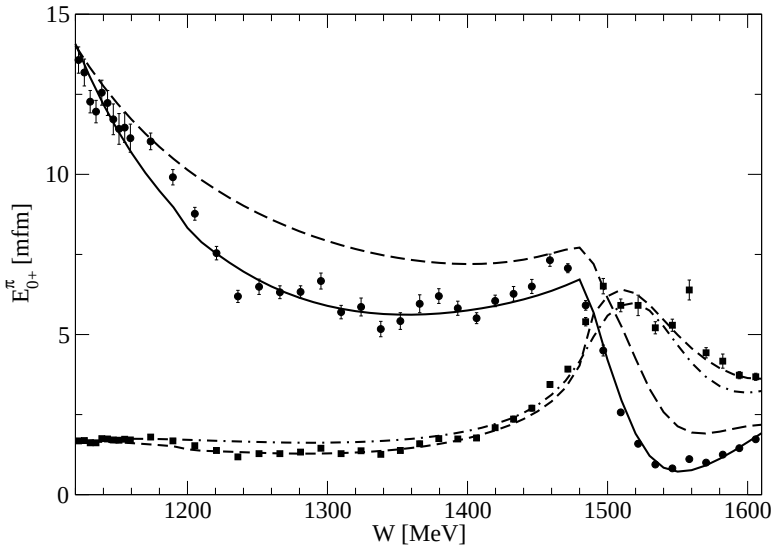
CNS Center for Nuclear Studies

Outline

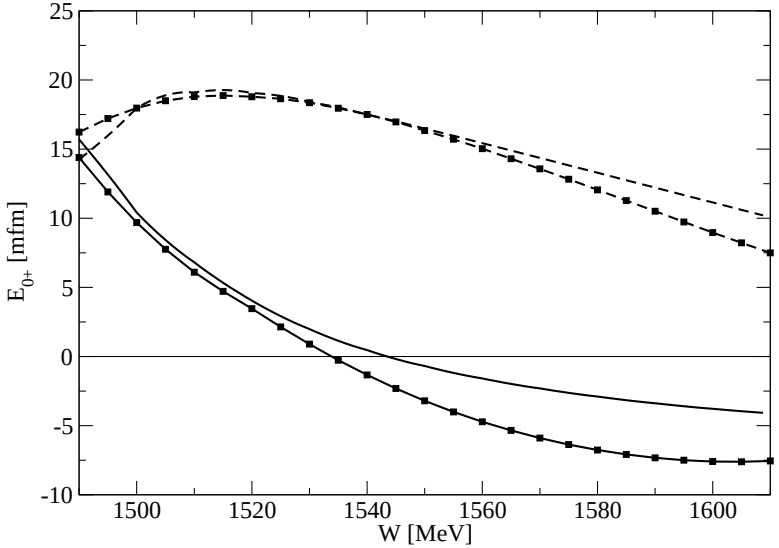
Center for Nuclear Studies

Comparison to MAID

E_{0+}^{π} SAID and MAID solutions



Refitting with MAID E_{0+}^{π} and same $|E_{0+}^{\eta}|$



Center for Nuclear Studies

Summary

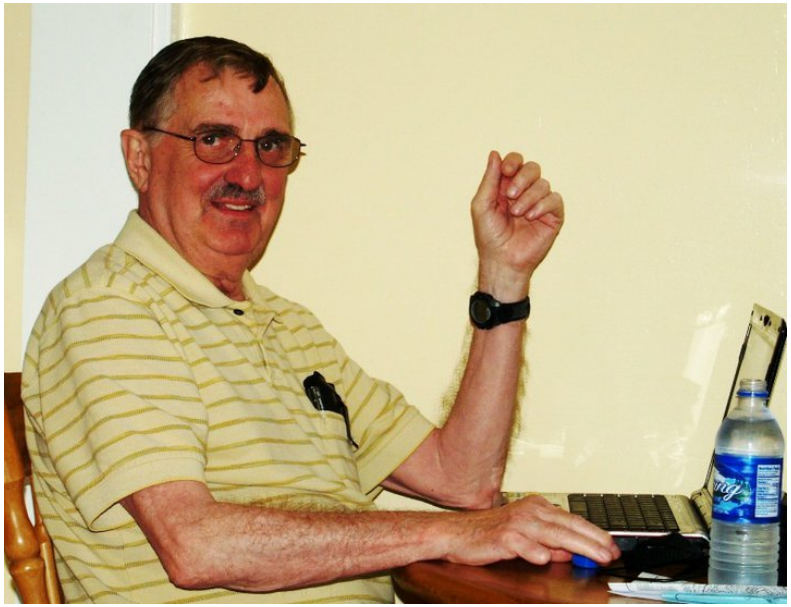
- Reviewed role of unitarity determining non-analyticities in the physical region, $w > m_i + m_t$ [almost complete]
- Related Chew-Mandelstam form to N/D approach $\rightarrow$ 'left-hand cut' neglected in C-M
- Performed simultaneous coupled-channel fit of $\eta-$photoproduction S_{11} multipole modulus, $|E^{\eta}_{0+}|$ and $\pi-$photoproduction amplitude, E^{π}_{0+}
- Current approach yields resonant E^{η}_{0+} phase $\rightarrow$ encourages us to pursue the C-M approach in fits to photoproduction observables (not amplitudes)

- Outlook
  1. Perform fit to $\pi-$photoproduction *data* using C-M form
  2. Perform simultaneous fit to $\pi-$ and $\eta-$photoproduction *data* using C-M form
  3. Perform simultaneous, global fit to $\pi N \rightarrow \pi N$, $\pi N \rightarrow \eta N$, $\gamma N \rightarrow \pi N$, $\gamma N \rightarrow \eta N$ using C-M form: **offers opportunity for precision electromagnetic data to 'back-constrain' hadronic amplitudes (some of which are very poorly known)**
  4. Generalize to correctly account for 'left-hand cuts'

CNS Center for Nuclear Studies

Dedication

To the memory of our friend and colleague, Dick Arndt, GWU Research Professor and Virginia Tech Emeritus Professor, who passed Saturday, April 10, 2010.



CNS
Center for Nuclear Studies